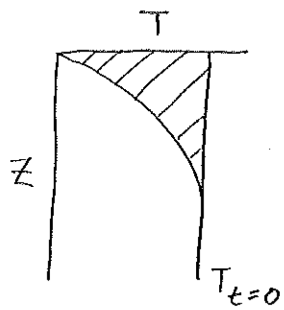
NOV 15, 2010
C-T LEE

DATING EARTH PROCESSES

HOW OLD IS THE EARTH?
WHEN DID EARTH DIFFERENTIATE?

EARLY APPROACH – LORD KELVIN TRIED TO CALCULATE AGE OF EARTH.

- ASSUMED INITIALLY EARTH WAS HOT AND HAS SINCE COOLED, THAT IS, $T_{t=0}$ IS KNOWN.



$$q = -k \frac{\Delta T}{\Delta z}$$ surface heat flux

HE ASSUMED CONDUCTIVELY COOLING
$\sim \Delta z \sim \sqrt{4Kt}$

THEN HE MEASURED q

$$t \sim \left(\frac{k\Delta T}{q}\right)^2 \frac{1}{4K}$$

$$t \sim \left(\frac{3\ W/mK \cdot 10^3 K}{\sim 5\times10^{-2}\ W/m^2}\right)^2 \cdot \frac{1}{4\cdot 10^{-6} m^2/s} \sim 100\ My$$

KELVIN DETERMINED THAT THE EARTH ~100 My OLD, WHICH WE KNOW TO BE WRONG.

- URBAN MYTH IS THAT HE GOT IT WRONG BECAUSE HE IGNORED RADIOACTIVITY.

$$q_{meas} = -k\frac{dT}{dz} + \dot{H}$$ ← Heat production

$$t \sim \left(\frac{k\Delta T}{q - \dot{H}}\right)^2 \frac{1}{4K}$$ accounting for $\dot{H}$ will give OLDER age, <u>BUT</u> IT WILL STILL BE WRONG.

- THE REAL REASON, KELVIN WAS WRONG, WAS HE IGNORED CONVECTION!!!

SO LET'S TURN TO RADIOMETRIC DATING TO GET A REAL AGE, NOT ONE BASED ON GEODYNAMIC MODELS.

RADIOACTIVE DECAY OCCURS IF NUCLEUS IS UNSTABLE

NUCLEUS → ISOTOPE (CONSTANT Z, variable N)

$^{Z+N}_{Z}T$ ↑ atomic number

Z = proton #

N = neutron #

$Z+N = A$ = atomic mass #

e.g. ^{187}Re and ^{185}Re

- NUCLEAR PARTICLES CAN STABILIZE IF THEY "BIND" TO EACH OTHER

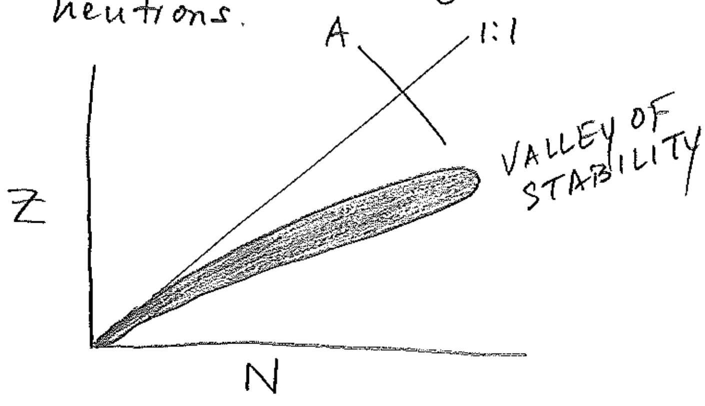
$$p + n = {}^{2}_{1}H + \text{energy}$$

energy is released (fusion)

$$\Delta m = m_{^2_1H} - m_p - m_n$$

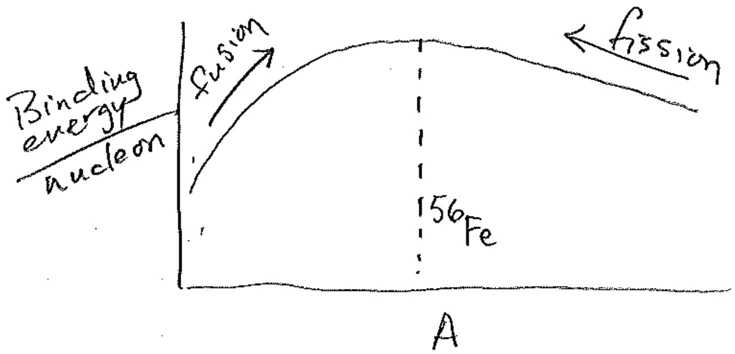
$$\Delta E = \Delta m c^2$$

BUT BECAUSE PROTONS REPEL $\left(\sim \frac{Z^2}{r^2} = F_{repulsion}\right)$ ONE CAN'T PUT TOO MANY NUCLEONS INTO A NUCLEUS. THIS REPULSION CAN BE COUNTERACTED BY ADDING NEUTRONS. WHEN Z gets too big, NEED even more neutrons.



- AT LOW MASS stable nuclides $Z \sim N$ or $Z \sim \frac{A}{2}$
- at hi mass $N > Z$

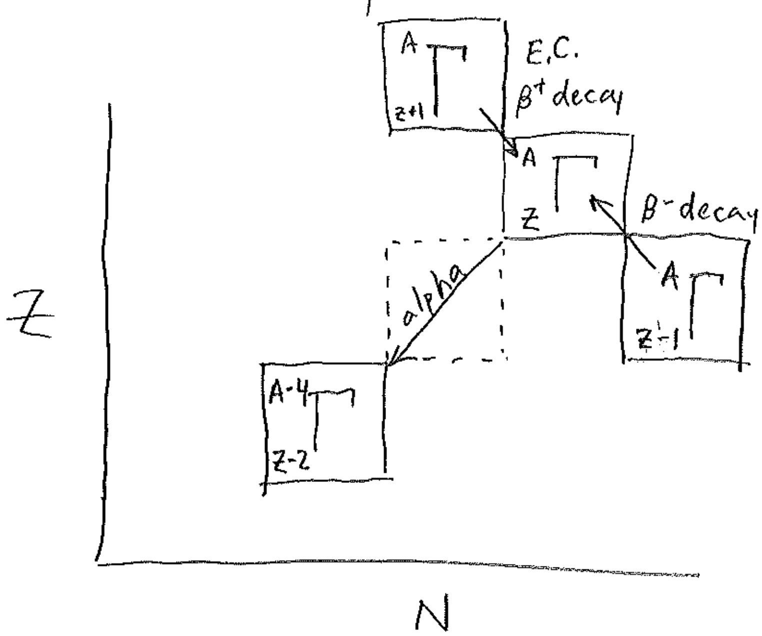
We also know that nuclear attractive forces decrease with distance of seperation. So although adding more N counteracts repulsion of protons, the nucleons are all further apart.



mass < ^{56}Fe, fusion

mass > ^{56}Fe, fission

RADIOACTIVE DECAYS



alpha decay $\quad ^{238}_{92}U = ^{234}_{90}Th + ^{4}_{2}He + \text{energy}$

β^- decay $\quad ^{87}_{37}Rb \rightarrow ^{87}_{38}Sr + \beta^- + \bar{\nu} + \text{energy}$

$n = p + e^-$

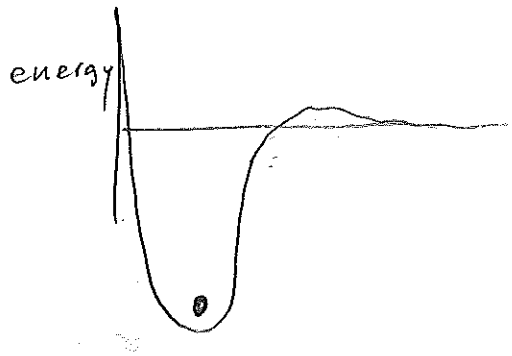
β^+ decay (positron) $\quad ^{40}_{19}K \rightarrow ^{40}_{18}Ar + \beta^+ + \nu + \text{energy}$

$p = n + e^+$

Electron Capture $\quad ^{40}_{19}K \rightarrow ^{40}_{18}Ar + \text{energy}$

$e^- + p = n$

Radioactive decay occurs if nucleus is unstable, but an energy barrier must be surpassed. This yields a rate constant for decay

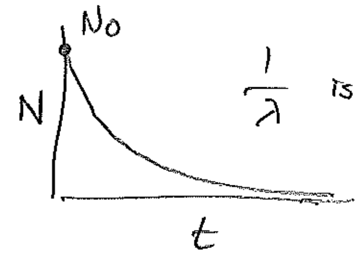


$N \rightarrow D$
parent daughter

$\frac{dN}{dt} \sim -N$ or $\frac{dN}{N\,dt} = \text{constant}$ } relative probability for decay

$$\frac{dN}{dt} = -\lambda N$$

$$N = N_0 e^{-\lambda t} \longrightarrow$$

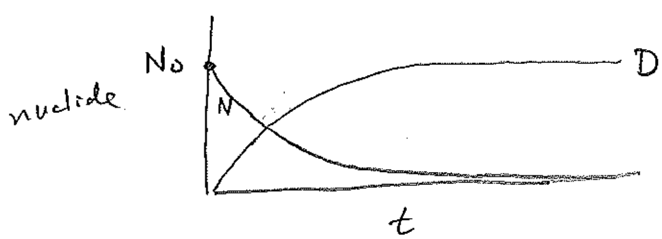


$\frac{1}{\lambda}$ is e-fold decay

$$t_{1/2} = \frac{1}{\lambda} \ln 2$$

$$N_0 = D + N$$

$$\hookrightarrow D = N(e^{\lambda t} - 1)$$

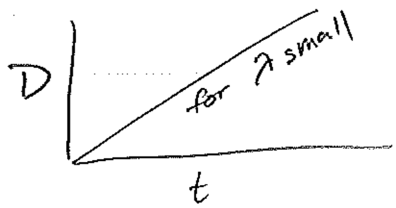


Radioactive decay systems useful for only ~8 half lives

Some useful approximations

$$e^{-\lambda t} \sim 1 - \lambda t \quad \text{if } \lambda \text{ is small}$$

so $$D \sim N \lambda t \quad \text{if } \lambda \text{ is small}$$

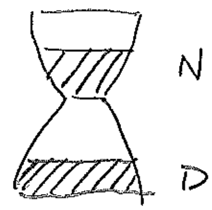


DATING WORKS AS FOLLOWS

$$D = N(e^{\lambda t} - 1)$$

UNKNOWN

measure   measure



HOUR GLASS
$N_0 = N + D$

BUT WHAT IF D_0 exists?

$$D = D_0 + N(e^{\lambda t} - 1)$$

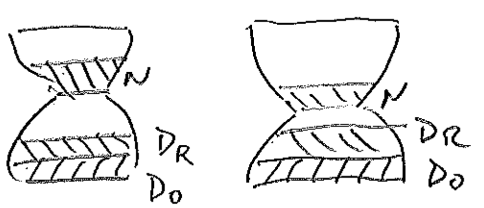
unknowns D_0 and t

so we need two equations at least

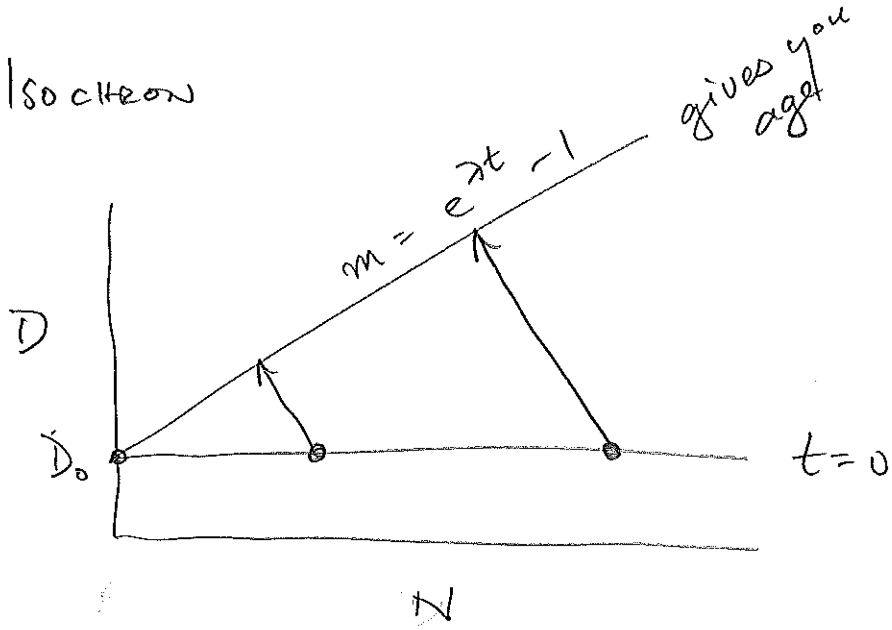
– 2 hourglasses

assuming t AND D_0 same.

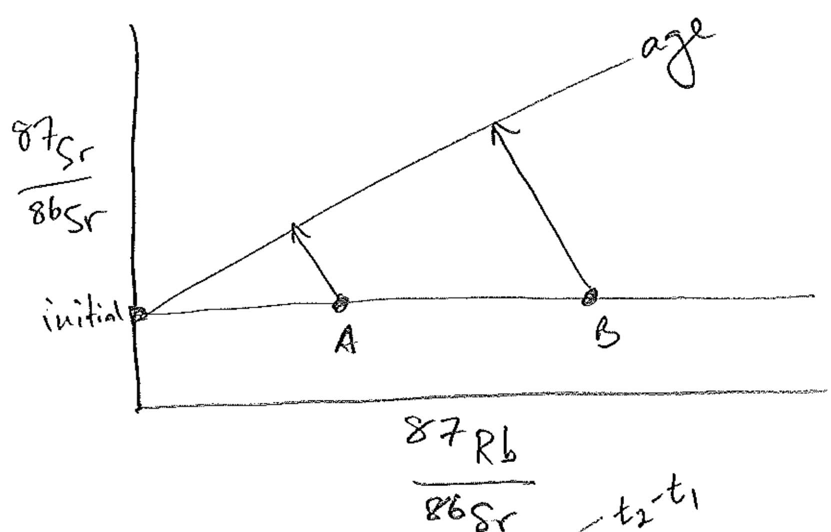
$t = 0$

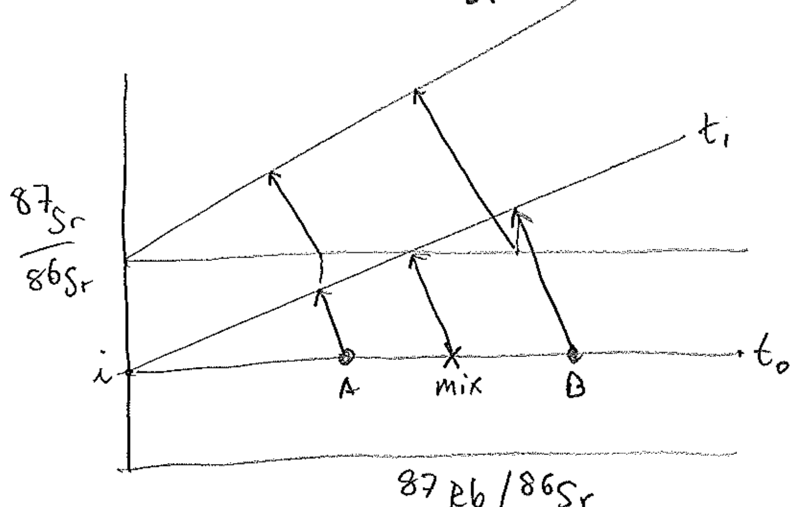


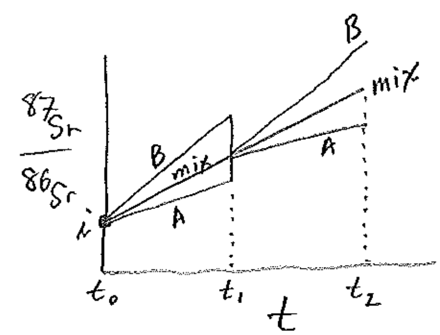
Isochron



typically we measure ratios w.r.t. a common, unradiogenic isotope, e.g.







t_0 = clock ticks
t_1 = homogenization
t_2 = final age
$t_2 - t_1$ = radiometric age

Keys for picking a good chronometer

- $t_{1/2}$ must be ~ to the timescales of your process of interest
- Parent · daughter ratio needs to be fractionated in the process you're interested in

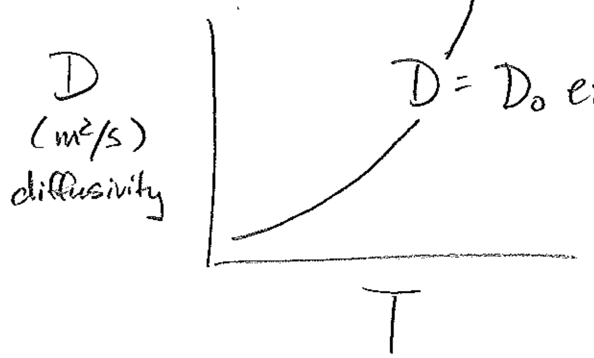
	$t_{1/2}$	process
$^{87}Rb \rightarrow {}^{87}Sr + \beta^-$	$t_{1/2}$ = 48.8 Gy	crust-mantle
$^{147}Sm \rightarrow {}^{143}Nd + \alpha$	106 Gy	crust-mantle
$^{187}Re \rightarrow {}^{187}Os + \beta^-$	42.3 Gy	core-mantle crust-mantle Sulfide-silicate
$^{238}U \rightarrow {}^{206}Pb + 8\,{}^4He$	4.47 Gy	core-mantle crust-mantle
$^{235}U \rightarrow {}^{207}Pb + 7\,{}^4He$	0.7 Gy	" "
$^{232}Th \rightarrow {}^{208}Pb + 6\,{}^4He$	14 Gy	" "
$^{176}Lu \rightarrow {}^{176}Hf + \beta^-$	37 Gy	crust-mantle
$^{190}Pt \rightarrow {}^{190}Os + \beta^-$	700 Gy	core - inner core
$^{40}K \rightarrow {}^{40}Ar$	1.25 Gy	Atmosphere-mantle

ISOCHRON DATING

- AGE SINCE SYSTEM WAS ISOTOPICALLY RESET
- RESETS WHEN T EXCEEDS CLOSURE TEMPERATURE

$> \sim T_c$, diffusive re-equilibration

$< T_c$, sub-systems closed to isotopic exchange



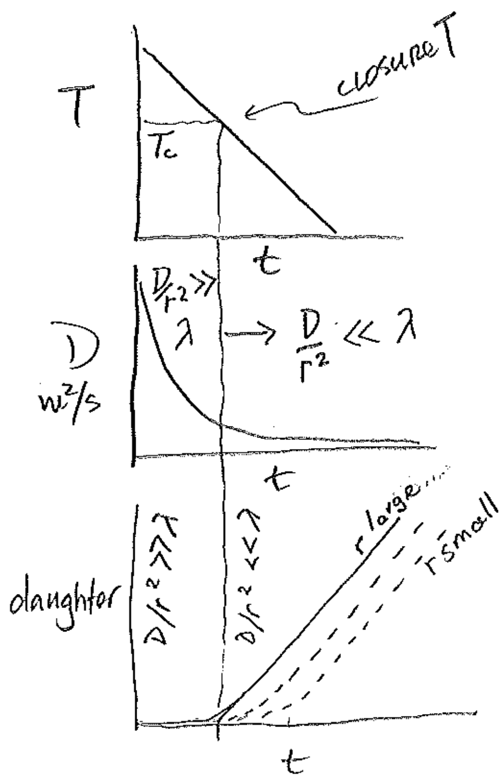
$$D = D_0 \exp(-E/RT)$$

Diffusion is T-controlled

diffusive lengthscale

$$r = \sqrt{4Dt}$$

timescale = $t = \dfrac{r^2}{4D}$ — if r large, remains closed; if r small, exchange is easy



$$\frac{\partial C}{\partial t} = -D\frac{\partial^2 C}{\partial x^2} + \lambda N$$

assume N is constant if λ is small.

C is daughter, N is parent

$\dfrac{D}{r^2} \ll \lambda \Rightarrow \dfrac{\partial C}{\partial t} \sim \lambda N$ CLOSED

↑radius of mineral

$\dfrac{D}{r^2} \gg \lambda \Rightarrow \dfrac{\partial C}{\partial t} = -D\dfrac{\partial^2 C}{\partial x^2}$ open

- closure occurs when you transition between these 2 regimes

Naturally occcurring radioactive isotopes

Isotope	% abundance	half life (Gy)	λ (y^{-1})	decay mode	decay product	Normalizing Isotope
^{235}U	0.7201	0.70	9.848E-10	7α & 4β-	^{207}Pb	^{204}Pb
^{40}K	0.01167	1.25	5.544E-10	β- (89.28%); EC & β+ (10.72)	^{40}Ca, ^{40}Ar	^{38}Ar
^{238}U	99.2743	4.47	1.551E-10	8α + 6β-	^{206}Pb	^{204}Pb
^{232}Th	100	14.01	4.948E-11	6α + 4β-	^{208}Pb	^{204}Pb
^{176}Lu	2.6	37.1	1.87E-11	β−	^{176}Hf	^{177}Hf
^{187}Re	63.93	42.3	1.64E-11	β−	^{187}Os	^{188}Os or ^{186}Os
^{87}Rb	27.8346	48.8	1.42E-11	β−	^{87}Sr	^{86}Sr
^{138}La	0.089	106	6.54E-12	β- / EC	^{138}Ce / ^{138}Ba	
^{147}Sm	15.07	106	6.54E-12	α	^{143}Nd	^{144}Nd
^{190}Pt	0.0127	700	9.90E-13	α	^{186}Os	^{188}Os

Short-lived isotopes

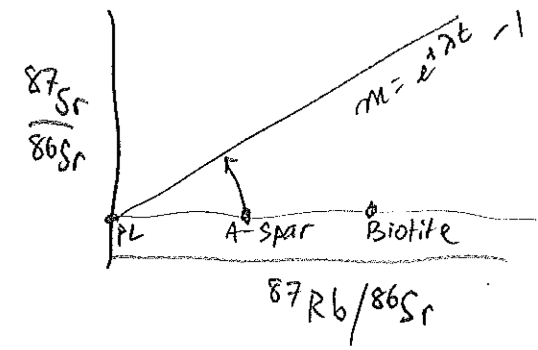
Isotope	half life (ky)	λ (y^{-1})	decay mode	decay product
^{32}P	0.0000392	1.77E+01	β−	^{32}S via ^{32}P
^{33}P	0.0000686	1.01E+01	β−	^{33}S
^{37}Ar	0.000096	7.23E+00	EC	^{37}Cl
^{7}Be	0.000146	4.75E+00	β-	^{7}Li
^{35}S	0.000238	2.91E+00	β-	^{35}Cl
^{22}Na	0.00260	2.67E-01	β+	^{22}Ne
^{3}H	0.0123	5.65E-02	β−	^{3}He
^{32}Si	0.170	4.08E-03	β−	^{32}S via ^{32}P
^{39}Ar	0.270	2.57E-03	β−	^{39}K
^{14}C	5.73	1.21E-04	β−	^{14}N
^{234}U	245	2.83E-06	α	^{230}Th
^{36}Cl	301	2.30E-06	β-	^{36}Ar
^{10}Be	1500	4.62E-07	EC	^{10}B

Extinct radioactive nuclides

Isotope	half life (My)	λ (y^{-1})	decay mode	decay product	
^{60}Fe	0.300	2.31E-06	β-	^{60}Ni via ^{60}Co	
^{26}Al	0.720	9.63E-07	β+; EC	^{26}Mg	^{24}Mg
^{53}Mn	3.75	1.85E-07	EC	^{53}Cr	^{52}Cr
^{107}Pd	6.48	1.07E-07	β−	^{107}Ag	
^{182}Hf	9.0	7.70E-08	β−	^{182}W	^{183}W
^{129}I	16.0	4.33E-08	β−	^{129}Xe	
^{236}U	23.9	2.90E-08	α	^{208}Pb via ^{232}Th	
^{92}Nb	34.7	2.00E-08	EC; β+	^{92}Zr	^{90}Zr
^{146}Sm	100	6.93E-09	α	^{142}Nd	^{144}Nd

Dating methods

Internal Isochrons for rocks



Pb-Pb isochrons

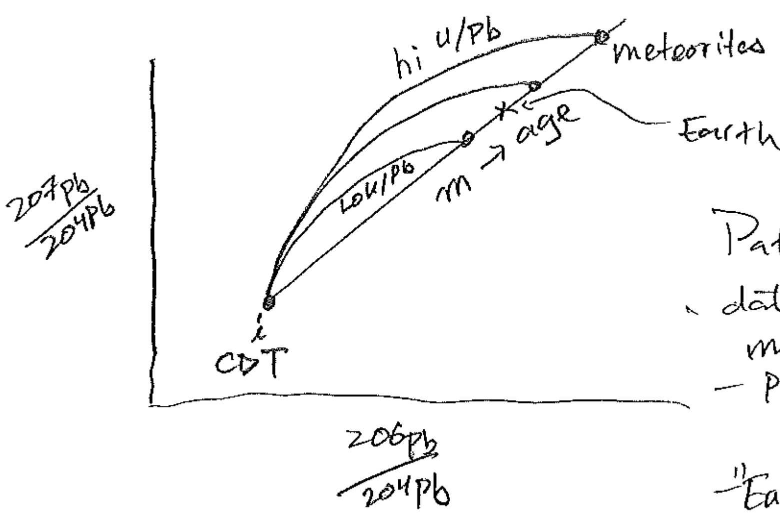
$^{238}U \rightarrow {}^{206}Pb$

$^{235}U \rightarrow {}^{207}Pb$

$$\frac{206}{204}Pb = \frac{206}{204}Pb_i + \frac{^{238}U}{^{204}Pb}\left(e^{\lambda_{238}t} - 1\right)$$

$$\frac{207}{204}Pb = \frac{207}{204}Pb_i + \frac{^{235}U}{^{204}Pb}\left(e^{\lambda_{235}t} - 1\right)$$

$$\Rightarrow \frac{\frac{206}{204}Pb - \frac{206}{204}Pb_i}{\frac{207}{204}Pb - \frac{207}{204}Pb_i} = \frac{^{238}U}{^{235}U}\underbrace{\frac{e^{\lambda_{238}t} - 1}{e^{\lambda_{235}t} - 1}}_{\text{age slope}}$$



Patterson, 1956
- date solar system using meteorites → 4.55 Gy
- Pb-Pb isochron
  geochron
- "Earth" falls on geochron

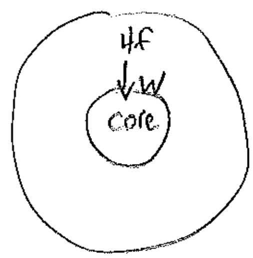
Short-lived nuclides.

What to do if parent has decayed all away

$^{182}Hf \rightarrow {}^{182}W + \beta^-$ $t_{1/2}$ 9 My

^{182}Hf is extinct $\rightarrow$ $D = N_0$

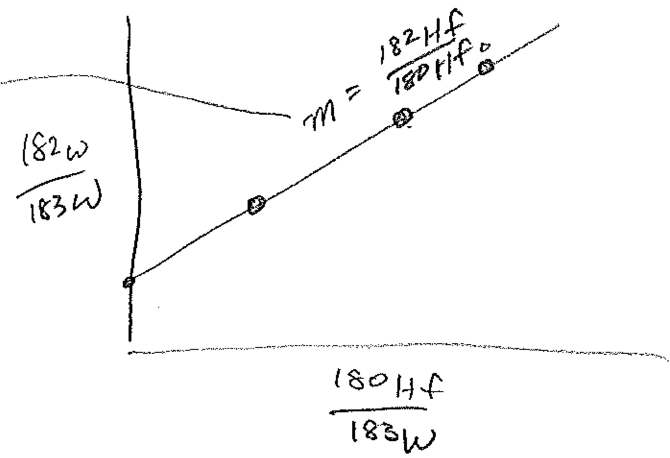
Good for core-mantle segregation



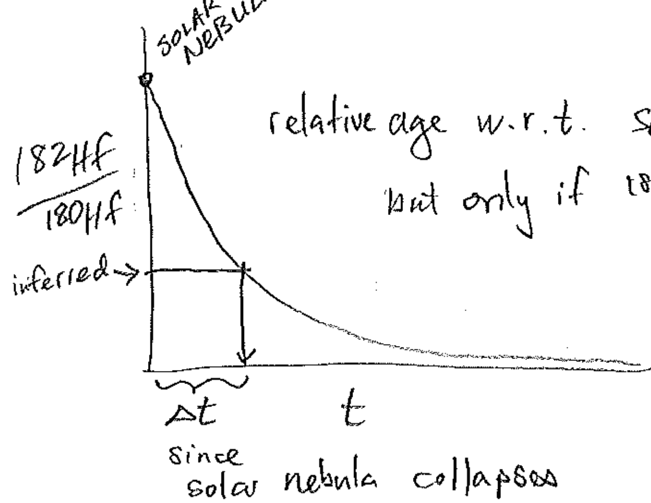
$$\frac{^{182}W}{^{183}W} = \frac{^{182}W_0}{^{183}W_0} + \frac{^{182}W_R}{^{183}W}$$

$$= \frac{^{182}W_0}{^{183}W_0} + \frac{^{182}Hf_0}{^{183}W_0}$$

$$\frac{^{182}W}{^{183}W} = \left(\frac{^{182}W}{^{183}W}\right)_0 + \frac{^{180}Hf}{^{183}W}\left(\frac{^{182}Hf}{^{180}Hf}\right)_0$$



- fossil ISOCHRON
- can't give t directly
- gives you $\frac{^{182}Hf}{^{180}Hf}$



relative age w.r.t. solar nebula can be determined

but only if $^{182}Hf/^{180}Hf$ of nebula is known!

For $^{182}Hf \rightarrow {}^{182}W$

↳ age of core f'm is ~30 My after solar nebula