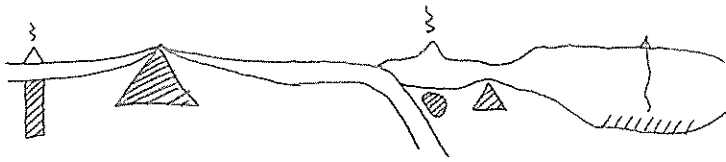


MELTING THE MANTLE

1. MOTIVATION



OBJECTIVES

$T, P, T_P, X, F, N_{0.8}$

IMPORTANT TO UNDERSTAND HOW + WHY MELTING OCCURS

- MELTING → CRUST FORMATION, PLANETARY DIFFERENTIATION
- MELTING → DEGASSING → ATMOSPHERE
- MELTING → REDISTRIBUTION OF TRACE ELEMENTS

MELTS CAN GIVE INSIGHT INTO

- THERMAL STATE OF EARTH'S INTERIOR
- COMPOSITION OF EARTH'S INTERIOR

KEY QUANTITIES WE WILL WANT ARE

T - temperature of melting

P - of melting

F - degree of melting (fraction of melting)

2. WHAT IS MELTING?

PHASE CHANGE FROM SOLID TO LIQUID

WHAT IS A SOLID ... LIQUID?

- SOLID MINERAL PHASE HAS A CRYSTALLINE STRUCTURE
LONG RANGE ORDER



LO ENTROPY

e.g. SiO_2 , WHERE ALL SiO_4^{4-} ARE BONDED

- LIQUID - SHORT RANGE ORDER



HI ENTROPY

e.g. molten SiO_2 ; some bonds are broken

* BE CAREFUL NOT TO CONFUSE THE DEFINITION OF A LIQUID WITH A RHEOLOGICAL FLUID.

3. MELTING OF A 1-COMPONENT (e.g. PURE) SYSTEM

ENERGY NEED TO MELT

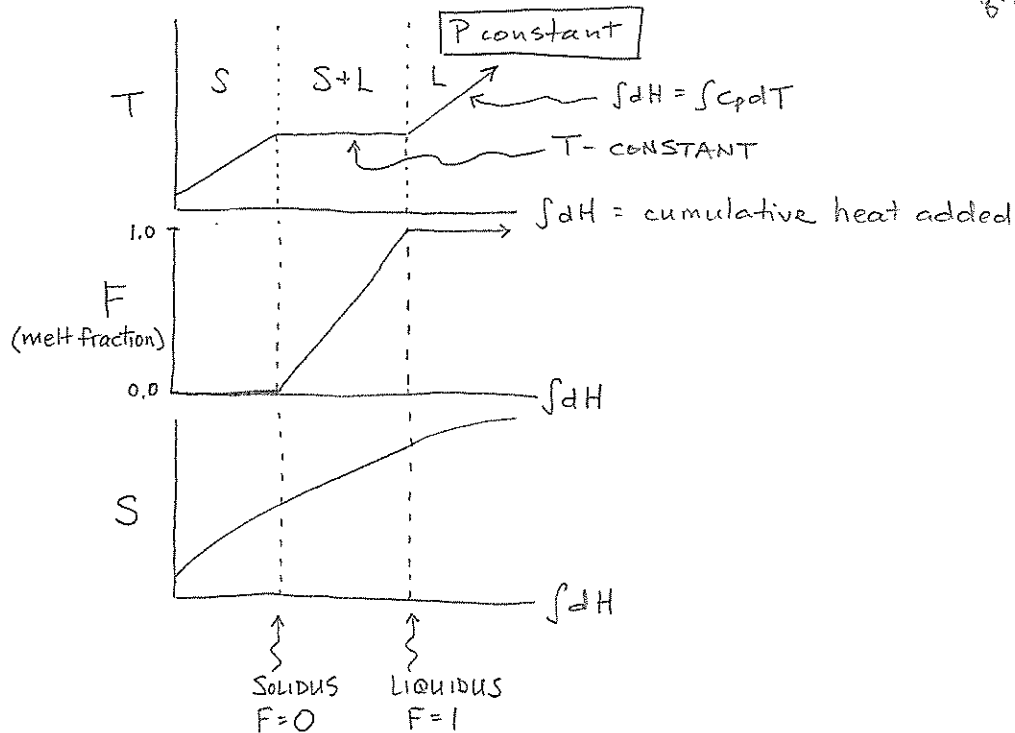
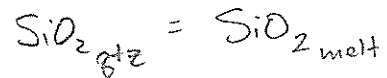
↳ HEAT OF FUSION ΔH_f

◦ THIS HEAT IS USED TO BREAK OR STRETCH BONDS

↳ ENTROPY INCREASES

$$\Delta S = \frac{\Delta H_f}{T_m}$$

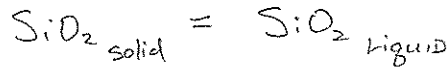
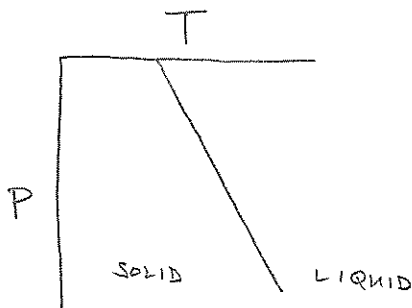
EXAMPLE,



* MELTING IS A PHASE CHANGE

* IN 1-COMPONENT SYSTEM, 2 PHASES COEXIST DURING MELTING AT CONSTANT P, T CANNOT CHANGE UNTIL ONE PHASE IS CONSUMED.

MELTING PHASE DIAGRAM IN P-T SPACE - 1 COMPONENT SYSTEM



$$\Delta V_{\text{rxn}} = V_{\text{Liq}} - V_{\text{Solid}} > 0$$

$$\Delta S_{\text{rxn}} = S_{\text{Liq}} - S_{\text{Solid}} > 0$$

SEE BELOW ↓

$$\left(\frac{dP}{dT}\right)_{\text{rxn}} = \frac{\Delta S_{\text{rxn}}}{\Delta V_{\text{rxn}}} > 0$$

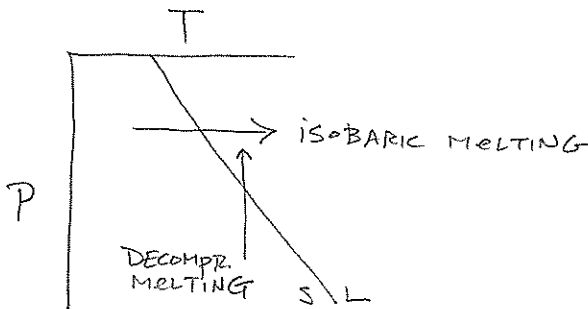
↑
CLAPEYRON SLOPE

IN GENERAL $\frac{dP}{dT} > 0$ FOR MELTING RXNS

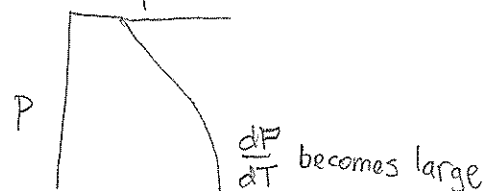
NOTE $\left(\frac{dP}{dT}\right)_{\text{rxn}} = \frac{\Delta H_f}{T_f \Delta V_f}$
↑ fusion

IF > 0 , THEN ENDOTHERMIC
(ASSUMING $\Delta V_f > 0$)

→ MELTING CAN OCCUR BY INCREASE IN T, DECREASE P



* NOTE: WITH INC P
 $\Delta V_{\text{melting}} \rightarrow 0$



* DERIVATION OF CLAPEYRON SLOPE

1st LAW

internal energy = heat + work

$$dU = q + w$$

$$dU = TdS - PdV$$

$$U(S, V) = TS - PV$$

DEFINE G (USING LEGENDRE TRANSF.)

$$G \equiv U - TS + PV = H - TS$$

$$G \equiv H - TS$$

Gibbs Free Energy ← enthalpy (heat + bond energy) entropy

$$dG = dU - TdS - SdT + VdP - PdV$$

combine w/ $dU = TdS - PdV$

$$dG = -SdT + VdP$$

FOR A RXN $d\Delta G_{\text{rxn}} = -\Delta S_{\text{rxn}}dT + \Delta V_{\text{rxn}}dP$

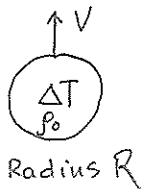
$d\Delta G_{\text{rxn}} = 0$ at equilibrium $\Rightarrow \frac{dP}{dT} = \frac{\Delta S_{\text{rxn}}}{\Delta V_{\text{rxn}}}$

DECOMPRESSION MELTING IS IMPORTANT IN EARTH

↑ ΔP large, ΔT small

How DOES DECOMPRESSION MELTING OCCUR?

* CONSIDER A HOT BLOB IN THE MANTLE.



$\Delta \rho = \rho_0 \alpha \Delta T$ thermal expansion \rightarrow density decrease

IF BUOYANT FORCE EXCEEDS VISCOUS RESISTANCE...

UPWARDS VELOCITY $V \sim \frac{\Delta \rho g R^2}{\eta}$

BLOB WANTS TO COOL

COOLING TIME $\sim \frac{R^2}{K}$

CHARACTERISTIC TRANSPORT TIME $\sim \frac{R}{V}$

WHEN $\frac{\text{COOLING } t}{\text{ASCENT } t} \gg 1$, RISES FASTER THAN IT COOLS

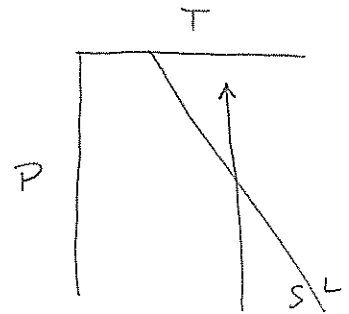
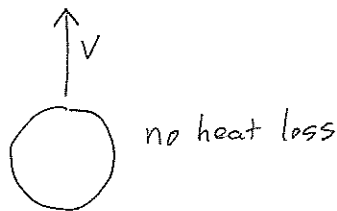
$$\frac{\Delta \rho g R^3}{\eta K} = \frac{\rho_0 \alpha g \Delta T R^3}{\eta K} = \text{Rayleigh number}$$

if $Ra > Ra_{cr} \sim 10^3$ CONVECTION

EARTH $Ra \sim 10^5 - 10^7$

CONCLUSION \rightarrow CONVECTION LEADS TO ADIABATIC UPWELLING

e.g. $q = 0$



ADIABATIC DECOMPRESSION

$$q=0$$

since $TdS \equiv q$, for a reversible process $\rightarrow dS=0$

ISENTROPIC PROCESS

$$\Delta U = q + w$$

$$\Delta U = TdS - PdV$$

isentropic

$$\Delta U = 0 - PdV$$

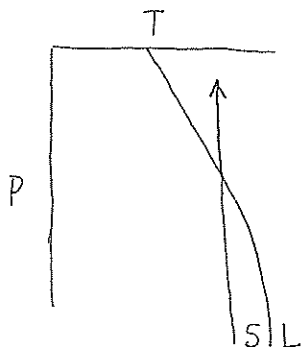
Isentropic decompression causes upwelling mantle to do work on surroundings, so internal energy drops (AND T drops)

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho C_p}$$

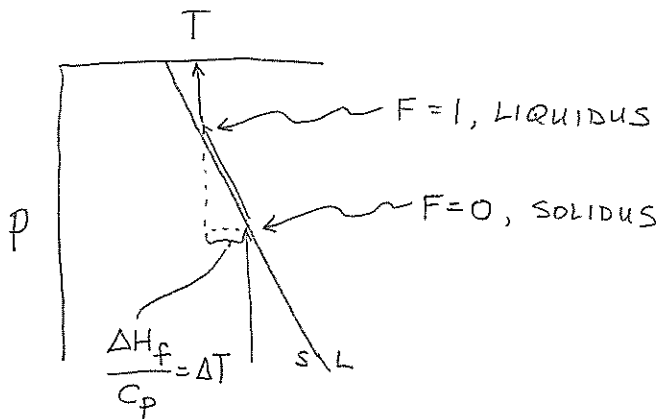
$$\text{OR } \left(\frac{\partial T}{\partial Z}\right)_S = \frac{\alpha g T}{C_p}$$

$$\text{SOLID } \left(\frac{\partial T}{\partial Z}\right)_S \sim 0.5^\circ \text{C/km}$$

Compare with SOLIDS $\sim 3-4^\circ \text{C/km}$ (NOTE THIS VARIES WITH DEPTH)

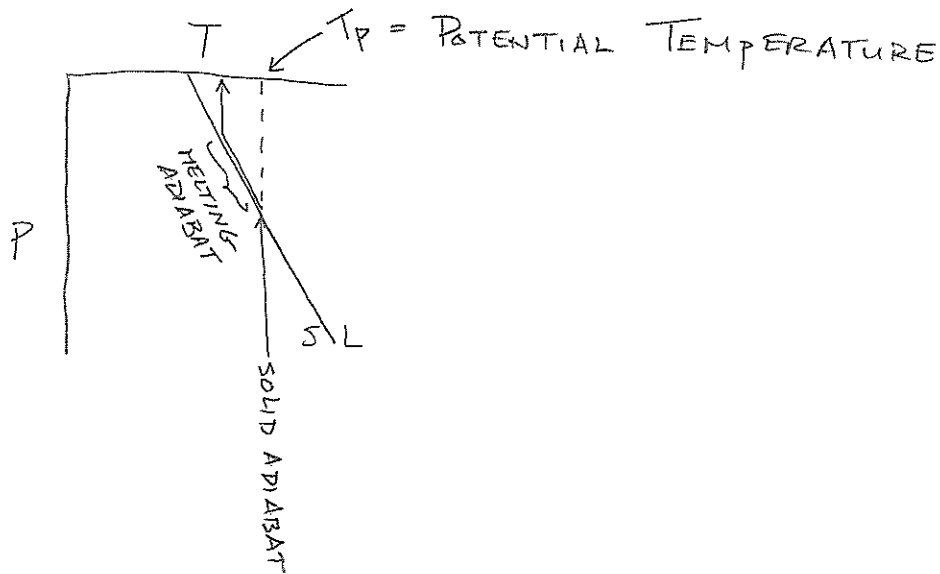


WHAT DOES ISENTROPIC MELTING LOOK LIKE IN A
1-COMPONENT SYSTEM



* AT CONSTANT P,
 $T_{\text{SOLIDUS}} = T_{\text{LIQUIDUS}}$
 * BUT NOT SO, DURING
 DECOMPRESSION

T DROPS DURING MELTING BECAUSE HEAT ABSORBED INTO
 PHASE CHANGE



MELTING IN MULTI-COMPONENT SYSTEM

BINARY EXAMPLE

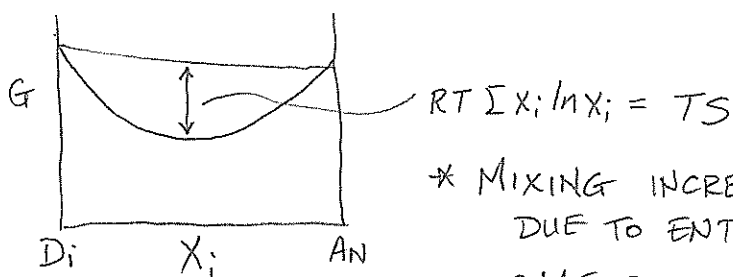
$$\bar{G}_{T,P} = \sum X_i \mu_i$$

$\bar{G}_{T,P}$: molar Gibbs free energy of phase
 X_i : mole fraction of component
 μ_i : chemical potential

$$\mu_i = \mu_i^\circ + RT \ln X_i \quad (\text{assume ideal mixing})$$

$$\bar{G}_{T,P} = \sum X_i \mu_i^\circ + RT \sum X_i \ln X_i$$

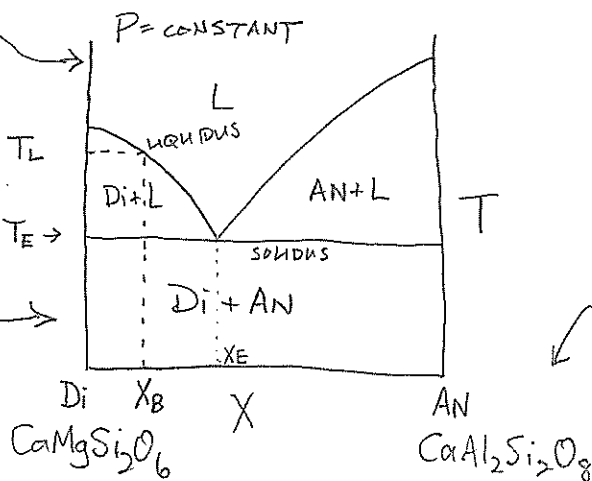
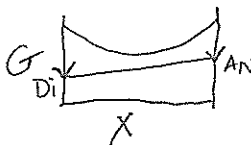
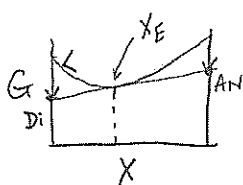
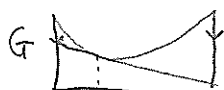
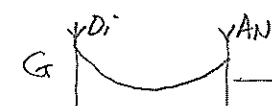
note similarity to $\Delta S_{\text{mixing}} = -R \sum X_i \ln X_i$



* MIXING INCREASES STABILITY OF A PHASE DUE TO ENTROPY INCREASE \rightarrow G DECREASE

• MELTS = SOLUTIONS

• MINERALS = IF SOLID SOLUTION, e.g. $(\text{Mg,Fe})_2\text{SiO}_4$



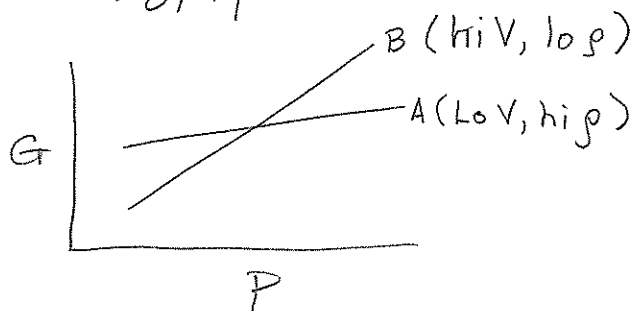
ANALOG OF A BASALT

RULES OF THUMB

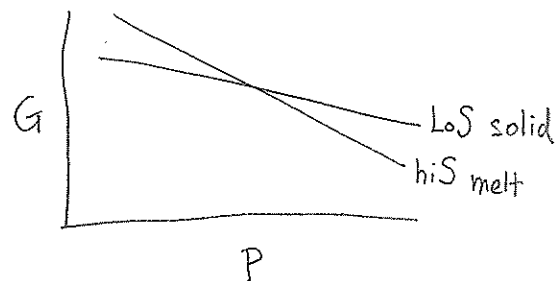
- ALL LIQUIDS AT THE ONSET OF MELTING HAVE SAME COMPOSITION (IN THIS, EUTECTIC COMP., X_E)
- $T_E = T_{\text{SOLIDUS}}$
- $T_{\text{LIQUIDUS}} \neq T_{\text{SOLIDUS}}$

BINARY XMP, EFFECT OF P

$$\left(\frac{\partial G}{\partial P}\right)_T = V$$

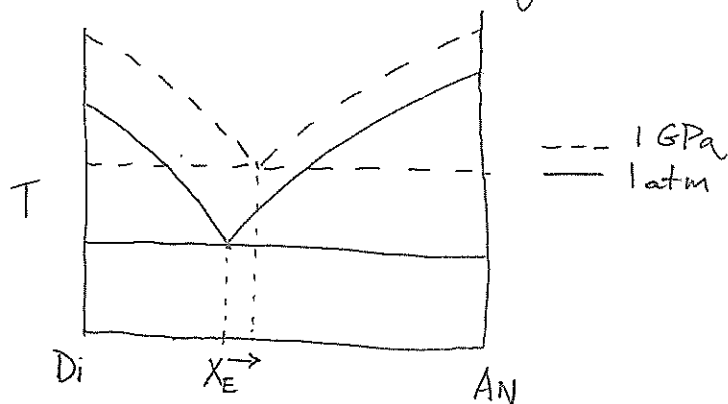


$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

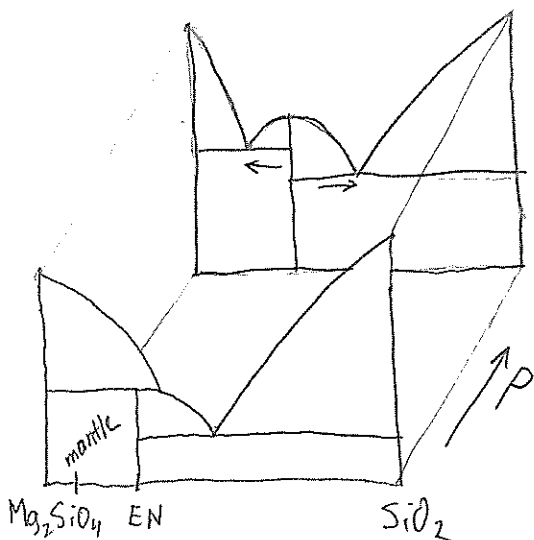


RULES OF THUMB

- * Hi P favors the stability of DENSE PHASES
 - ∴ HARDER TO MELT AT HIGHER P, SO T_E RISES
 - ∴ X_E , EUTECTIC COMPOSITION, MOVES AWAY FROM hiP PHASE (i.e. phase field for hiP phase expands)
- * Hi T favors hi entropy phases
melts require hi T.



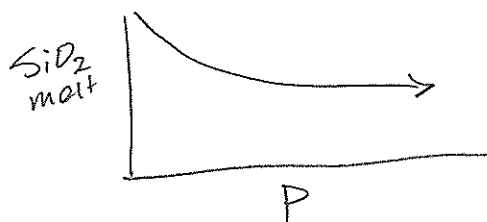
BASALT ANALOG



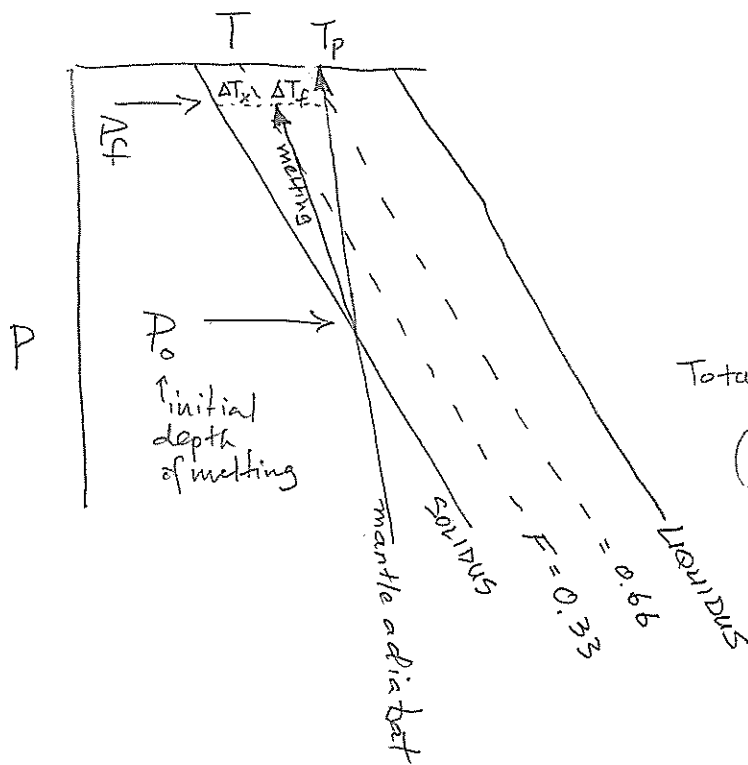
MANTLE PERIDOTITE ANALOG

Mg_2SiO_4 OLIVINE
ENSTATITE $Mg_2Si_2O_6$

$P \uparrow$ SiO_2 in melt \downarrow



ISENTROPIC DECOMPRESSION MELTING IN MULTI-COMP. SPACE



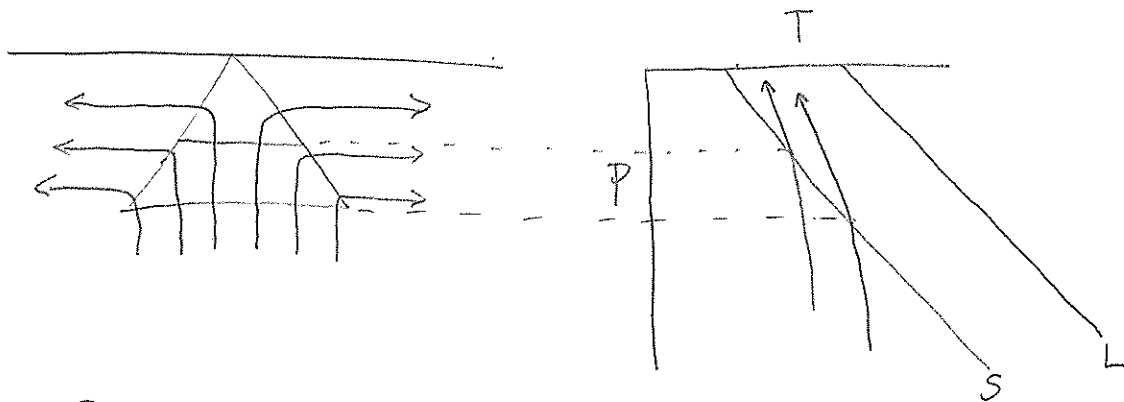
$$\Delta T_f = \frac{F \Delta H_f}{C_p}$$

$$\Delta T_x = F \frac{dT}{F}$$

Total heat content

$$(P_0 - P_f) \left(\frac{\partial T}{\partial P} \right)_{\text{solidus}} - (P_0 - P_f) \left(\frac{\partial T}{\partial P} \right)_{\text{adiab}}$$

MELTING AT A RIDGE

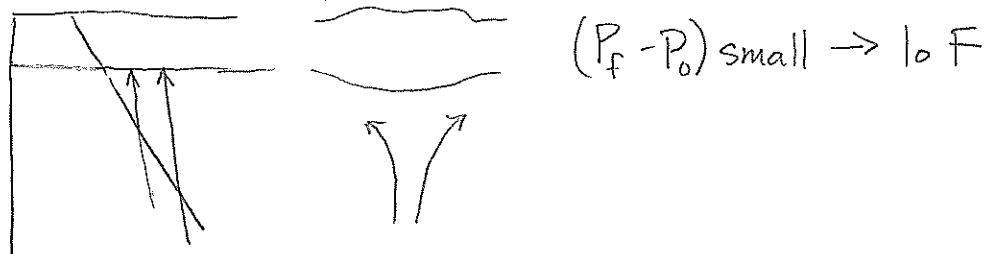


RULES OF THUMB

$$h_i T_p \rightarrow h_i F, h_i P_0$$

$$L_o T_p \rightarrow L_o F, L_o P_0$$

IF THERE IS A LITHOSPHERIC CAP



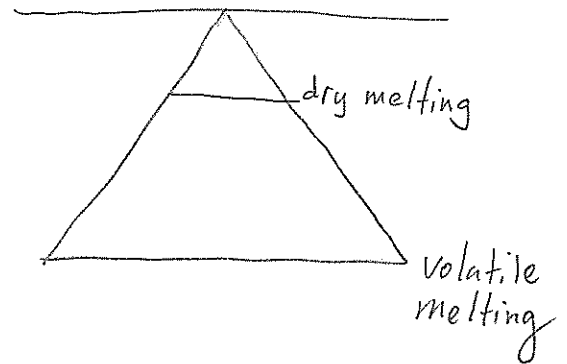
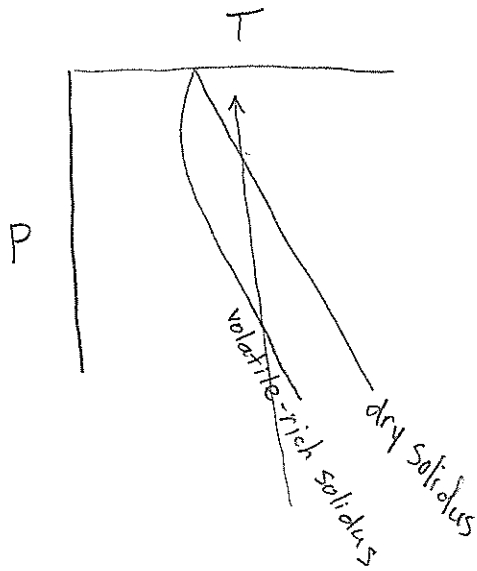
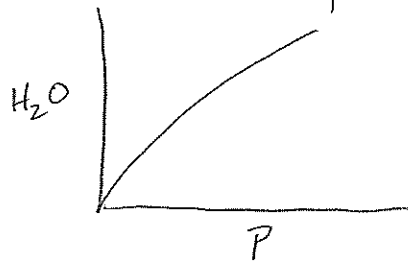
EFFECT OF VOLATILES

SOLIDUS IS DEPRESSED IF VOLATILES ARE SOLUBLE IN MELT.

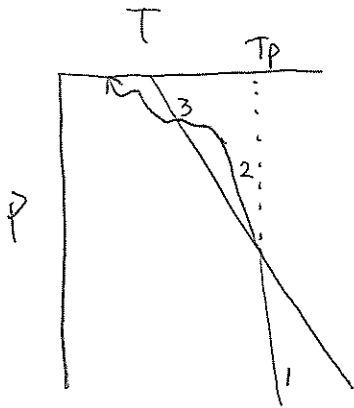
H_2O , CO_2 SOLUBLE IN SILICATE MELT

CH_4 , C NOT SOLUBLE SO REDOX MATTERS

BECAUSE SOLUBILITY VARIES AS P , FREEZING POINT DEPRESSION IS SMALLER AT LO P .



GOAL IS TO GET T_p BECAUSE THIS GIVES THERMAL STATE



- 1 - solid adiabatic
- 2 - melting adiabatic
- 3 - non-adiabatic cooling

IF WE KNOW T , F OR P OF PRIMARY MAGMA THEN WE CAN SAY SOMETHING ABOUT T_p .

PRIMARY = IN EQUILIBRIUM W/ MANTLE SOURCE

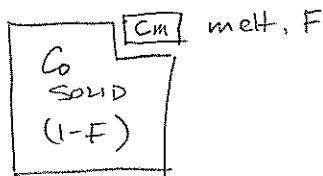
EVOLVED = NOT IN EQ W/ SOURCE DUE TO FRACTIONAL CRYSTALLIZATION OR CONTAMINATION

How DO WE GET T_p ?

F , T , P OF MELTING?

F CAN BE DETERMINED FROM INCOMPATIBLE TRACE ELEMENTS

SIMPLE APPROACH -
BATCH EQUILIBRIUM MELTING



PARTITION COEFFICIENT

$$D = \frac{C_{\text{SOLID}}}{C_{\text{MELT}}}$$

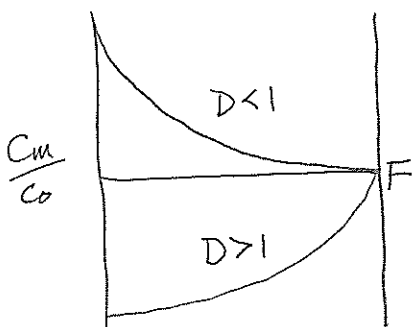
> 1 COMPATIBLE

< 1 INCOMPATIBLE

$$C_o = C_m F + C_s (1-F)$$

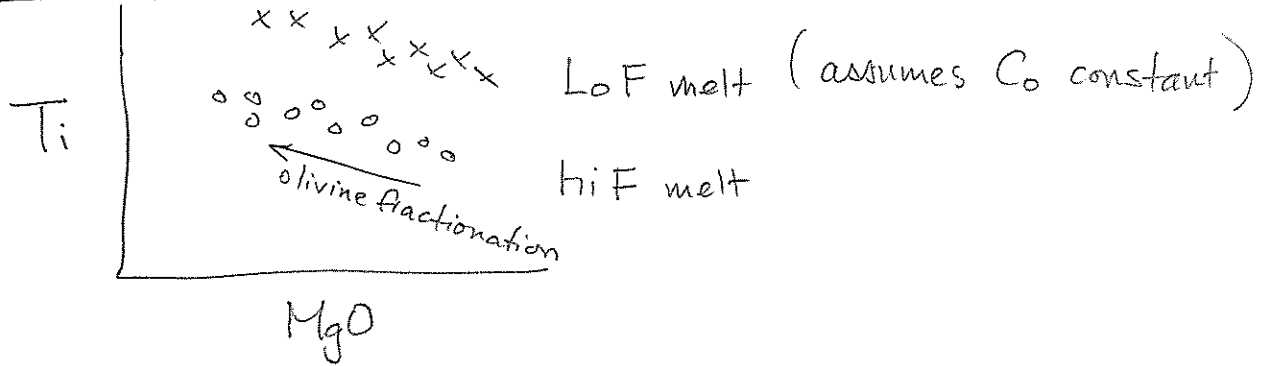
$$\frac{C_m}{C_o} = \frac{1}{D + F(1-D)}$$

← melt enrichment factor

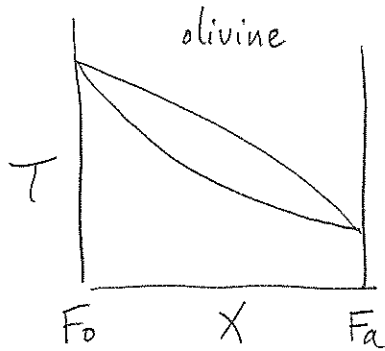


general rules: IF $D=0$ $\frac{C_m}{C_o} = \frac{1}{F}$
IF $F=0$ $\frac{C_m}{C_o} = \frac{1}{D}$

application

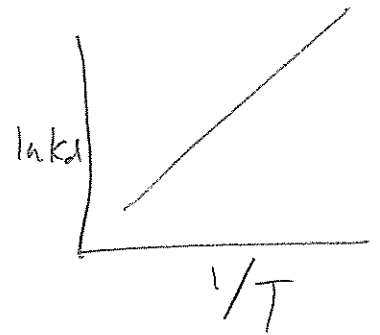


TEMPERATURE - THERMOMETER



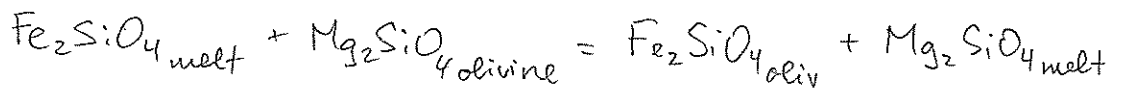
$$K_d = \frac{MgO_{olivine}}{MgO_{liquid}}$$

$$\ln K_d = \frac{a}{T} + b$$

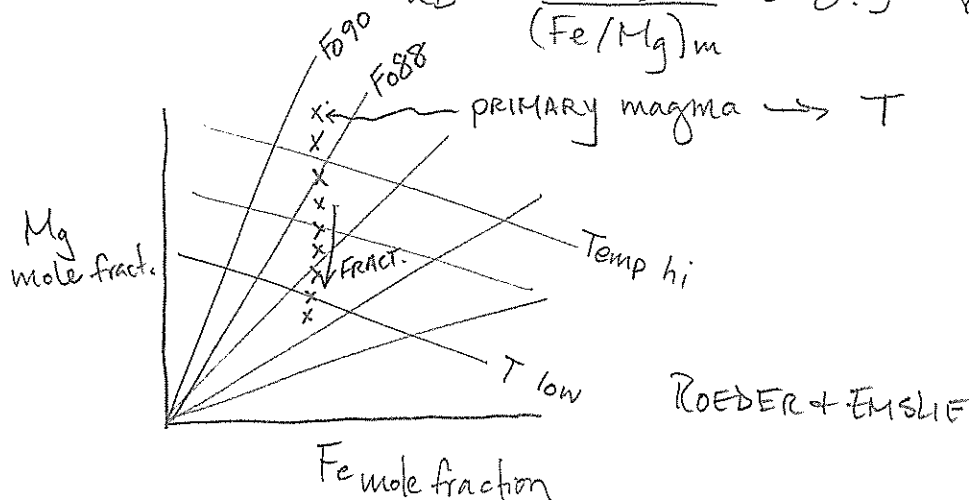


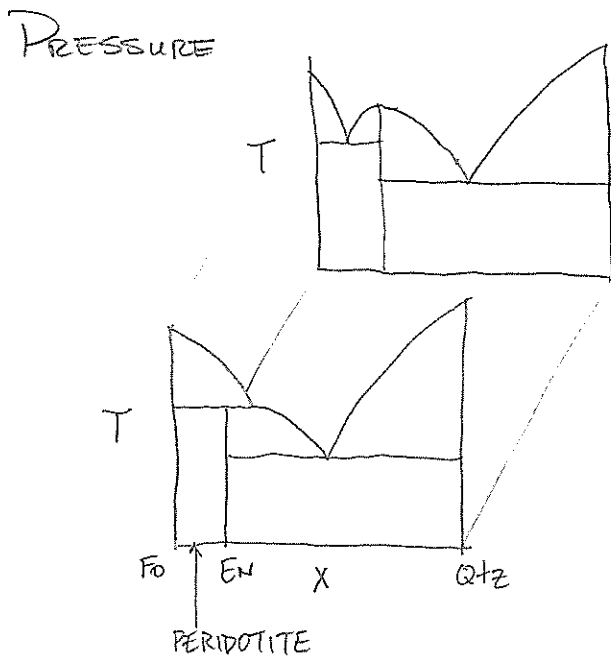
FROM MgO content of melt, you can get T
 * High MgO → hi T (as long as olivine saturated)

IN DETAIL

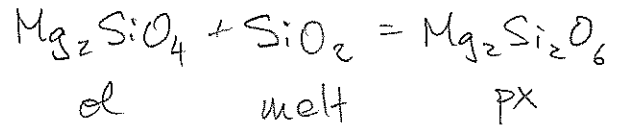


$$K_D = \frac{(Fe/Mg)_{ol}}{(Fe/Mg)_m} \sim 0.3 \text{ relatively constant with } T, P$$





A PRESSURE-SENSITIVE RXN



$$K = \frac{a_{px}^{En}}{a_{ol}^{Fo} a_{melt}^{SiO_2}}$$

$$\frac{a_{px}^{En}}{a_{ol}^{Fo}} \sim \text{constant}$$

$$K \sim \frac{1}{a_{melt}^{SiO_2}}$$

OLIV+OPX SYSTEM
BUFFERS SiO_2

RULES OF THUMB

* Low $\text{SiO}_2 \rightarrow$ hi P (assuming you have a primary magma)

CAVEATS/COMPLICATIONS

- MANTLE SOURCE VARIABLE IN TERMS OF MAJOR COMPOSITION
- PRIMARY MAGMA NOT KNOWN
- MAGMAS ARE AGGREGATES OR REACTED PRODUCTS.